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Using Transition Systems to Formalise Ideas from Vedānta

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Abstract:
Vedānta is one of the oldest philosophical systems. While there are many detailed commentaries on Vedānta, there are very few mathematical descriptions of the different concepts developed there. This article shows how ideas from theoretical computer science can be used to explain Vedānta. The standard ideas of transition systems and modal logic are used to develop a formal description for the different ideas in Vedānta. The generality of the formalism is illustrated via a number of examples including *saṃsāra*, Patañjali's *Yogasūtra*s, *karma*, the three *avasthā*s from the *Māṇḍūkya Upaniṣad* and the key difference between *advaita* and *dvaita* in relation to *mokṣa*.
Keywords: Vedānta, *mokṣa*, transition system, Brahman.

1. Introduction

The *Upaniṣad*s, also called Vedānta, [9], [25], move away from the purely ritualistic worship of God that is present in the earlier section of the Vedas. The *Upaniṣad*s are viewed as one of the earliest philosophical texts and many of them pre-date Buddhist thought. These writings cover a variety of topics including the origins of the universe, what happens after death, what is the root cause of our experiences, and "Who am I?" or "What is my true self". They also wish to answer the question of what is eternally true and what is changeable.

Because many of the *Upaniṣad*s are terse, the core ideas in the *Upaniṣad*s are expanded in different writings including detailed commentaries. These include, the *Bhṛguvalli* [29, 21], the *Pañcadaśī* [1], the *Yoga-Vāsiṣṭha* [2] and the *Vivekacūḍāmaṇi* [17]. More recent works (e.g. [31]) provide a high-level summary of many of these concepts. They all explain the notion of *Brahman*, the ultimate entity who is above the Vedic Gods, based on the descriptions in the *Kena* and other *Upaniṣad*s [4]. Technically, everything owes its existence to *Brahman* and *Brahman* is the sole "cause" of everything. In other words,

if *Brahman* did not exist, nothing would exist. The writings also go on to argue that one's true self, i.e., the answer to the question "Who am I?" is related to *Brahman*. Many of the *Upaniṣad*s also state that *Brahman* is not an object of knowledge. So *Brahman* cannot be known using conventional means of acquiring knowledge. *Brahman* is a pure subject and the basis of all experiences [6].

Explanations related to the universe and human existence is often based on *karma* or the law related to action and its consequences. *Karma*, in terms of the consequences that need to be bourne, is associated with an individual's *jīva*, which could be described as the individual's soul. It is the *jīva* that "carries" the karma. It is the presence of karma that leads to *saṃsāra* which is the cycle of birth and death. *Mokṣa*, which is salvation or freedom from *saṃsāra*, is the aim of all spiritual seekers [16]. *Mokṣa* is not only getting out of *saṃsāra* but it is also being one with the divine. The different interpretations of Vedānta have slightly different definitions of being one with the divine. In *advaita* one's true self is identical to *Brahman*. Therefore, *mokṣa* in *advaita* is realising that one is *Brahman*. This realisation is not just bookish knowledge. It is about how one interacts with everything and everyone in the world. In *dvaita*, one attains *mokṣa* when one reaches the abode of God. *Dvaita* claims that one cannot merge or become one with *Brahman*. This is because the *jīva*'s soul is different from *Brahman*.

All interpretations of *Vedānta* are based on the showing that one's true self is not the body or the mind. To explain this line of thought, these writings introduce different concepts including

- the five body sheaths or the *Pañcakosha* [21],
- the three types of bodies or *śarīra*s [24],
- *mithyā* which is loosely translated as illusion or what is unreal,
- *sat* which is the opposite of *mithyā* and thus loosely translated as real,
- *anirvacanīya* or the one that cannot be described because of limitations in language and
- the *neti-neti* (or apophatic) style of reasoning [28].

While the above ideas are developed at length using natural language, they are still subject to different interpretations. In this article, we describe a formalism that can, in principle, capture the semantics of all these different concepts. This is based using a labelled transition system [20, 12], and modal logic related to knowledge [10]. The process formalism used here can describe a variety of state based dynamic behaviours based on changes to the current state. This formalism can also be used to describe concurrent behaviours. While we do not focus on concurrency in this article, it is important that concurrency can be supported. Concurrency is needed to define how different entities evolve independent of each other as well how different entities can interact together.

The main purpose of the formalisation is to present a framework where all the key concepts from Vedānta can be defined in a precise way. The intention is that the formalisation will provide the basis for further discussions including distinguishing the different interpretations of the same concept. The usefulness of the formalisation is illustrated via a number of simple examples. These examples are not necessarily complete, in that they do not completely describe all the concepts in Vedānta and associated texts. They only illustrate the use of formalism to describe some of the key concepts. The examples also show how the formal descriptions can be used to characterise the different interpretations of Vedānta or comparing Vedānta with other schools of thought [19]. Such detailed comparisons are beyond the scope of this paper. Similarly, we do not focus on the logical aspects present in Vedānta-related epistemology or concrete reasoning systems (such as Nyāya [7]). Such descriptions can be found elsewhere, e.g. [5].

In short, the aim of the paper is to give a formal semantics to ideas from Vedānta. Towards this we use the idea of transition systems and logic of knowledge that are common in computer science. The key aspects of the formalism are first described in Section 2; the examples that use the formal

notation are described in Section 3. Based on the examples in Section 3, a high-level system view is presented in Section 4.

2. Formalisation

In this section we explain the notation that underpins the formalism used to describe the key concepts. The formalisation is based on the following building blocks.

- An infinite set of states $\mathbb{S}$ with a distinguished element called B. Here $B \in \mathbb{S}$ represents *Brahman*.
- This set of states is *partitioned* into many subsets. That is, the intersection of each distinct subsets is empty. Each subset is typically written as S_j for a particular entity j. For example, j can be a *jīva* which will be expanded in Section 3.1. Therefore, for different entities j_1 and j_2, $\mathbb{S}_{j_1} \cap \mathbb{S}_{j_2} = \emptyset$.
- An infinite set of actions $\mathbb{A}$ that represents the actions that can be performed.
- An infinite set of properties $\mathbb{P}$ that can be associated with each state except B. B has no properties axiomatically. A number of properties may hold in any given state. Thus the set of properties for any given state will be a subset of $\mathbb{P}$.

Some of the descriptions in Vedānta are in terms of knowledge. That is, Vedānta outlines what can be known, who can know what, etc. Here we focus on subset of the techniques outlined in the book by Fagin et al. [10]. For the purposes of this article, we define a binary relation K to represent knowledge. Elements of this relation belong to P as they are used to describe logical predicates. So we can describe what is known and by whom in any given state. For instance, if $\mathbb{K}(a, b)(s)$ holds in a given state, we can conclude that entity a knows b in state s. We can also have $\mathbb{K}(a, \mathbb{K}(b, c))(s)$ which indicates in state s, a knows that b knows c. As per the *Dṛg-Dṛśya-Viveka* [23], we cannot have $\mathbb{K}(o, o)$ as the object is different from the knower of the object. But we can have a chain of knowledge relations. For example, $\mathbb{K}(o_2, o_1)$, $\mathbb{K}(o_3, o_2)$, and $\mathbb{K}(o_4, o_3)$ are all possible knowledge relations. That is, o_2 knows o_1, o_3 knows o_2 and o_4 knows o_3. So an object can know other objects but not itself. Disallowing self-reference prevents logical inconsistencies.

- As everything other than *Brahman* changes, it is natural to capture change as transitions between states. So we define a set of transitions as pairs of states labelled with action(s). This is written as $s \xrightarrow{a} s'$ where $a \in \mathbb{A}$. This indicates that one can move from state s to state s' by performing action a.

Sometimes (mainly for notational convenience) we write $s \xrightarrow{\phi} s'$ where ϕ represents the change in properties as a result of performing some action a. Here the focus is not on the action but on the change in properties corresponding with the change in state. Thus $\phi \subseteq \mathbb{P}$.

- We use B as the initial state for the transition system. This will be used to capture the fact that everything starts with *Brahman*. This indicates that without *Brahman* nothing can exist.

The above formalism suffices for the main concepts we wish to illustrate via examples. The generality of this formalism has been illustrated in other research where different types of systems are described. The formalism presented in this article is more concrete than the other characterisations of Vedānta. For instance, the logic we are using here is relatively simple. More complex modal logics (e.g. [27]) can be incorporated in this framework without major effort. Similarly, Corazza [8] uses axiomatic set theory to define consciousness (an important aspect of Vedānta) but does not handle state transitions that occur in the material universe. State-based systems can be used to describe consciousness [13].

3. Examples of Reasoning

In this section we present a few examples to illustrate some of the concepts developed in Vedānta. The examples presented here are chosen to cover a number of diverse topics to illustrate the generality of the approach.

3.1. Brahman *and* Jīva

Vedānta states that everything exists because of *Brahman*. This is captured by the requirement that for every state *s*, one can find a path from *B* to *s*. As *B* is the initial state nothing in the system can be obtained without *Brahman B*. This formalism answers the question what was there before *Brahman*. As *Brahman* is the initial state, the question of having something before *Brahman* does not arise. Otherwise, the entity before *Brahman* would be the initial state.

Furthermore, no entity can know *Brahman* and *Brahman* has no properties. This means that in all states s and for all entities *e* including B, we have $\neg\mathbb{K}(e,B)(s)$ and $\neg\mathbb{K}(e,\mathbb{K})(s)$. That is, *Brahman* cannot know *Brahman* and it is not possible to know the relation $\mathbb{K}$. These requirements are added to avoid potential logical contradictions. The fact that *Brahman* has no properties holds by definition.

The next idea we consider is that of *jīva* or soul that has not attained *mokṣa* or salvation. This captures the idea of living entities in this world. The reason for dividing the set of states into a set of disjoint states is that each subset represents the behaviour associated with a particular *jīva*. Formally, for each (say j), we can identify a set of states $\mathbb{S}_j \subset \mathbb{S}$ and as described earlier for different j_1 and j_2 the $\mathbb{S}_{j_1} \cap \mathbb{S}_{j_2} = \emptyset$. These properties state that the evolution of each *jīva* occurs within its own state space. Every *jīva* can interact with other *jīva*s and this is illustrated in Section 3.4.

Every *jīva* that is alive has a *sthūla* (gross), *sūkṣma* (subtle) and *kāraṇa* (causal) *śarīra* (bodies) [24]. To capture this, we define properties called 'has*Sthūla*', 'has*Sūkṣma*' and 'has*Kāraṇa*'. Formally, {has*Sthūla*, has*Sūkṣma*, has*Kāraṇa*} $\subset \mathbb{P}$.

These properties are associated with the layers of the *jīva* and can be used to characterise, both being embodied and being dead. In any state where all three properties hold, the *jīva* is said to be embodied. When the body dies, the *sthūla śarīra* ceases to exist. Thus death is characterised where 'has*Sthūla*' does not hold but 'has*Sūkṣma*' and 'has*Kāraṇa*' hold. Formally, death can be an abbreviation for the formula '¬has*Sthūla* ∧ has*Sūkṣma* ∧ has*Kāraṇa*'.

These three properties can also be used to characterise *pralaya* or dissolution of the universe. When *pralaya* occurs, all *jīva*s have only their *kāraṇa śarīra*. Both the *sthūla śarīra* and the *sūkṣma śarīra* cease to exist. So a state *s* can be said to represent *pralaya* when for all *jīva*s the property '¬has*Sthūla* ∧ has*Sūkṣma* ∧ has*Kāraṇa*' holds.

The properties associated with the *śarīra*s allow us to impose consistency checks. For example, if 'has*Sthūla*' holds, then both 'has*Sūkṣma*' and 'has*Kāraṇa*' need to hold. Similarly, if 'has*Sūkṣma*' holds, then 'has*Kāraṇa*' has to hold.

Given this characterisation of a *jīva*, reincarnation is now a sequence of transitions from a state where the property associated with death holds to a state where the *jīva* acquires a new body, i.e., a *sthūla śarīra*. Consider the sequence of transitions $s_1 \overset{death}{\rightarrow} s_2 \overset{born}{\rightarrow} s_3$. Here s_1 and s_3 will belong to the same state partition associated with a specific *jīva*. In states s_1 and s_3 the property 'has*Sthūla*' holds while in state s_2 the property 'has*Sthūla*' will not hold. It is not essential for states s_1 and s_3 to be identical. The difference in s_1 and s_3 could be due to the change in karma (see Section 3.3) that is

associated with those states.

3.2. Free Will

In some theories, God acts as a controller also called *antaryāmī*. But this does not necessarily mean that all the *jīva*s have a pre-determined life. The *jīva*s have free will. God's role in free will can be defined in terms of the set of transitions that are available at each state. That is, the set of transitions represents the options one has at any given point. Each *jīva* can choose one of these options based on its current tendencies, thinking etc., capturing the semantics of free will. That is, free will is having choice to select possible behaviours at any given state.

An example is shown in Figure 1. Assume that in state s_1 three choices are possible. God might decide, for whatever reason, that in this state the option to perform b_1 should not permitted. Thus the *jīva* still has choice to perform either action a_1 or c_1 but state s_3 is not reachable from s_1. If the *jīva* chooses a_1, the states s_5, s_8 and s_9 are potentially reachable. That is, God is not constraining any behaviour from s_2 and s_5. However, if the *jīva* chooses c_1, the only possible move is to state s_6 via action c_2 because action c_3 is blocked by God.

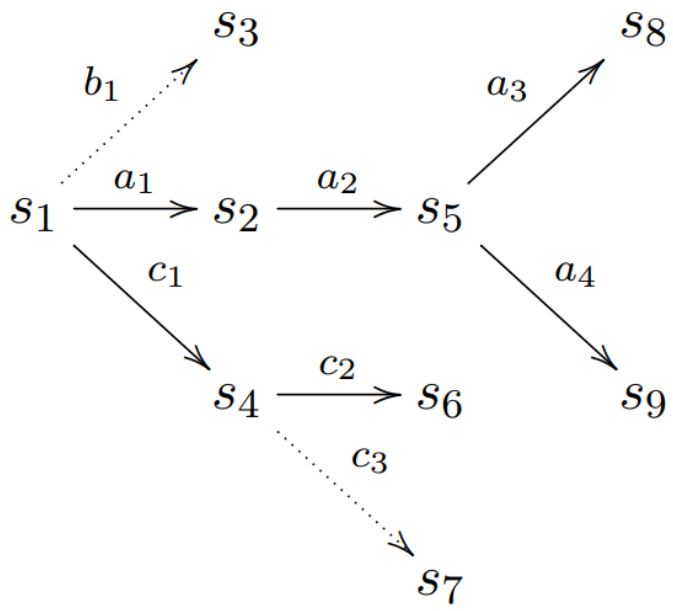

Figure 1: Free Will

The example shows that behaviour of the *jīva* is not pre-ordained. Both God, via making options available, and the *jīva*'s free will by choosing the option that is made available have a role in deciding what happens in the future.

Such control of behaviour could occur via assigning specific *karma* values (developed in Section 3.3) to each state. So one's past or current *karma* could enable or disable certain transitions. Therefore, the value associated with *karma* can be used to encode either the enabling or inhibition of certain actions.

Overall, God or the principle of *karma* can act as controller (as in discrete control systems [26]) where certain actions are disabled while all the *jīva*s are like the environment in system theory. That is, the behaviour of all the *jīva*s is unpredictable as they have free will. They are free to choose from the available list of actions. But unlike a safe controller, not all unsafe behaviours are necessarily blocked by God. The chosen behaviour, be it good or bad, is left to the individual.

To capture this formally, we define a class of properties ''godAllows(α)' for every action α (i.e., $\alpha \in \mathbb{A}$). This can be used to describe aspects of the transition system in Figure 1 as follows.

In state s_1 the property 'godAllows(a_1) $\wedge$ godAllows(c_1) $\wedge \neg$ godAllows(b_1)' holds.
In state s_4 the property 'godAllows(c_2) $\wedge \neg$ godAllows(c_3)' holds.
In state s_5 the property 'godAllows(a_3) $\wedge$ godAllows(a_4)' holds.

3.3. Karma *and* Mokṣa

Karma can also be encoded in this system as a specific class of property. Changes in *karma* are captured via changes in the set of properties between the starting and ending state. For example, let us assume for the sake of simplicity that the property of *karma* is denoted as a pair of integer values representing good and bad karma (i.e., $\langle g, b \rangle$). If one does a good action but it is not without dispassion the in the new state will be $\langle g', b' \rangle$ where $g' > g$ and $b' \leq b$. The usual understanding is that b will not change but it is possible to have a theory where a good action offsets a part of past bad *karma*; hence b can decrease. Similarly, a bad action will result in the value of b increasing (g may or may not increase) and a dispassionate action will not change the values associated with *karma*.

Normally, *mokṣa* occurs when one's accumulated karma is zero or $\langle 0,0 \rangle$. *Mokṣa* at one level is simple and it occurs when the transition from a state to *B* is taken. That is, one has reached the end goal of spirituality, namely, "being one with *Brahman*". Such a transition only occurs when the karma is $\langle 0,0 \rangle$.

Such transitions are not sufficient as they do not handle the notion of *jīvan mukta* [18]. A *jīvan mukta* is one who has realised *Brahman* but is still living, i.e., has a body. A *jīvan mukta* can be represented as a state where all the three *śarīra*'s exist and there is no *karma* (i.e., *karma* is $\langle 0,0 \rangle$). To capture the semantics of *jīvan mukta*, for all such states where they are alive, the only possible move is to a state where the body is dead and then both the *sūkṣma śarīra* and the *kāraṇa śarīra* disappear leading to *Brahman*.

Technically, we have to split karma into *sañcita karma* or the accumulated *karma* during this birth, *prārabdha karma* or the *karma* that is associated with one's birth and *āgami karma* that is the result of current actions. That is, we are refining the pair of integers can be split into '*sañcitaKarma*', '*prārabdhaKarma*' and '*āgamiKarma*'. This does not require any change to the basic framework. Only the encoding of *karma* has to change from a pair into three pairs. The above description of change in karma via current actions performed will apply only to the *āgami karma*. When we say a *jīvan mukta* has no *karma* it applies only to the *sañcita* and *āgami karma*. The *prārabdha karma* will disappear only at the time of death.

All transitions in the living world made by a *jīvan mukta* must keep the absence of *sañcita* and *āgami karma* invariant. That is, all transitions for a *jīvan mukta* from a state where there is no *sañcita karma* has to be to a state where there is no *sañcita karma*. Some aspects of this is captured by the transitions shown in Figure 2. Here actions *a* and *b* are performed when the *jīvan mukta* is living. Such an invariant applies only after the *jīva* has become a *jīvan mukta*. Otherwise, the *jīva* will continue to

accumulate *sañcita karma*.

Thus the property 'has*Sthūla*' holds in states s_1, s_2 and s_3. Also, '*sañcitaKarma*(s_1) = *sañcitaKarma*(s_2) = *sañcitaKarma*(s_3)' and '*āgamiKarma*(s_1) = *āgamiKarma*(s_2) = *āgamiKarma*(s_3)' are true. When the body associated with the *jīvan mukta* dies, the *jīvan mukta* enters the state s_4 where 'has*Sthūla*' does not hold. Now the only possible transition is to *Brahman*. This transition is called *videha mukti* in the literature (i.e., *mukti* achieved without a body or *mokṣa* after death). The absence of any other transition is shown using ×. Formally, *videha mukti* occurs in state (say s_4) where $\neg\exists\ s_5 \neq B, a: (s_4 \xrightarrow{a} s_5)$. That is, there is no action or state (other than *Brahman*) that the state s_4 can evolve to.

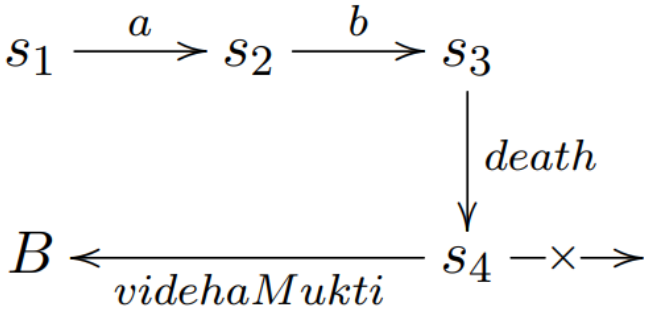

Figure 2: *Jīvan Mukta*

3.4. Interaction with Others and Joint Behaviour

Thus far we have outlined the behaviour of a particular *jīva* without any reference to other *jīva*s. In reality, each *jīva* interacts with other *jīva*s. To capture this, we define an interaction relation which will contain all possible interactions. This requires an extension to the basic formalisation which considered each state transition in isolation.

Formally, interaction is represented by a set by a set $\mathbb{I}$. This can be formally defined in terms of subsets of the relation →. Each interaction is a set of transitions from different *jīva*s. For example, the set $\{s \xrightarrow{\alpha} s', t \xrightarrow{\beta} t', u \xrightarrow{\gamma} u'\}$ represents an interaction between three *jīva*s. It describes the situation where the in states s, t, u interact with each other and move to the states s', t', u' respectively. The actions α, β, and γ need to be performed by the individual *jīva*s for the interaction to occur. We can impose a consistency requirement on elements in $\mathbb{I}$. We require that the partitions that contains s, t and u respectively are all mutually disjoint. That is, interaction occurs only between different *jīva*s. So s, t and u have to belong to sets associated with three distinct *jīva*s.

We now give a simple example that uses the above formalism. The example describes the incident from the *Mahābhārata* where Bhīma hits Duryodhana's thighs. This can be seen as an interaction between Bhīma, Duryodhana, Sanjaya (who was narrating the incident) and Dhṛtarāṣṭra (who was listening to Sanjaya). This can be represented by the following four transitions operating together.

Bhīma: $b \xrightarrow{hit} b'$ where hit represents hitting Duryodhana's thighs.

Duryodhana: $du \xrightarrow{gotHit} du'$ where $gotHit$ represents getting hit on the thighs by Bhīma.

Sanjaya: $s \xrightarrow{describe} s'$ where $describe$ is Sanjaya narrating the incident.

Dhṛtarāṣṭra: $dh \overset{emotions}{\rightarrow} dh'$ where the action $emotions$ represents both feeling sad at Duryodhana's plight and feeling angry at the Pandavās. That is, the state dh' associated with Dhṛtarāṣṭra denotes him feeling sad at the impending loss of his son, Duryodhana, and also angry at the Pandavās for inflicting damage to his children.

Without explicitly considering interaction, one can state that *hit* and *gotHit* have to occur in the same step. Other than this notion of "simultaneity", none of the previous descriptions, say related to properties such as karma need to change. One only needs to define what is the outcome of the joint behaviour. While joint behaviours requires the participation of multiple *jīva*s, the outcome for each *jīva* in terms of the resulting state is defined individually. Therefore, the idea of associating properties with states needs no change. For example, if aspects of the interaction are unethical, the performers' negative *karma* will increase and the *karma* of the one who suffered could reduce. Specifically, the result of this interaction could be the following.

- Bhīma accumulates some negative *karma* (for violating the rules of war). This change in karma will be reflected in the property associated with the state *b'*.
- Duryodhana who has become mortally wounded, has undergone suffering and will have some reduction in his negative *karma*. As he is not yet dead, he still has his *sthūla śarīra*. Similar to *b'*, the change in Duryodhana's *karma* will be reflected in the property associated with *du'*.
- Dhṛtarāṣṭra has also suffered and his negative *karma* will be reduced. The reduction would depend on the level of mental anguish offset with his emotions desiring revenge.
- As Sanjaya is just an observer and is not affected by the above actions, there will be no change in *karma* for Sanjaya.

3.5. Aspects from the Yogasūtras

Patañjali's *Yogasūtra*s [3], have had a huge impact on Vedantic thinking especially the ideas related to meditation and controlling the mind. In this section we describe two related concepts from the *Yogasūtra*s. The first is a wandering mind, where in a given state one cannot focus and the second is a calm mind which is not affected by the behaviour of others.

To describe a wandering mind, we first define a set of actions (say $T \subseteq \mathbb{A}$) to represent one's thoughts. A mind is wandering in a given state (say s) when different actions from T lead to different states. For example, let a and b belong to T, and $s \overset{a}{\rightarrow} s_a$ and $s \overset{b}{\rightarrow} s_b$ be the two possible transitions where s_a and s_b are different states. If the mind is focussed on only one thought (say on a), the transition $s \overset{b}{\rightarrow} s_b$ will not be taken. Otherwise in state s, the mind is wondering which action (a or b) to perform.

In a way, this is related to free will. That is, certain transitions will not occur. In the case of a focused mind, the control in terms of which transitions are generated (i.e., elements of *T*) and which are not taken is exercised by the *jīva*. Here, unlike in the case of free will, God has no role.

In general one's mind is less wandering in state s_1 than in state s_2 if the cardinality of the set $\{s_a | s_1 \overset{a}{\rightarrow} s_a, a \in T\}$ is less than the cardinality of the $\{s_a | s_2 \overset{a}{\rightarrow} s_a, a \in T\}$. That is from s_1 there are fewer options than from s_2. Because the number of possible choices the mind has to consider (i.e., actions from *T*) in state s_1 is less than the number of possible choices in state s_2, one can conclude that the mind in state s_1 is not wandering as much as from s_2. An transition of form $s_1 \overset{a}{\rightarrow} s_1$ indicates that the action a has no effect on s_1 and also represents a non-wandering mind, for the action a. Here the

thought a occurs but has not change the *jīva*'s state.

In the above formalisation, the set of actions in T are atomic. We can enhance T with actions that represent the thought arising, the thought being extinguished as well as actually performing the action that arose. This only increases the granularity of the possible transition system.

Towards defining a calm mind, we consider transitions in $\mathbb{I}$ because we wish to measure the effect of external effects on one's mind. One's mind is calm in state s_1 with respect to a particular thought $a \in T$ when for every set I in $\mathbb{I}$, there is only state s_2 where $s_1 \xrightarrow{a} s_2$ irrespective of the other elements in I. That is, the behaviours of *other entities* have *no effect* on the behaviour from the state s_1. This definition allows change from state s_1; but that would be based purely on the thought process of the *jīva* associated with state s_1.

As an example consider the two interactions $\{s \xrightarrow{a} s_1,\ t \xrightarrow{b} t_1\}$ and $\{s \xrightarrow{a} s_2,\ u \xrightarrow{c} u_1\}$ where $s_1 \neq s_2$. Here the *jīva*'s behaviour in state s on the action a is influenced by the actions b or c resulting in different consequences. Such a behaviour represents a mind that is not calm. The mind is reacting to what others are doing (b or c in this case). However, if s_1 and s_2 were identical, the mind can be said to be calm in this particular situation as it effectively ignores the influence actions b and c.

The above definition of a calm mind can also be used to define *kṣhānti* (forbearance) where one is equipoised in all circumstances [14]. That is, the behaviour (i.e., transition) chosen by people exhibiting *kṣhānti* will not depend on the action of the others around them.

3.6. Dream State and Entities in a Dream

The *Māṇḍūkya Upaniṣad* [22] discusses how the three *avasthā*s of waking, dreaming and deep sleep are all different from Brahman. Here we show how the basic structure of the set of states and knowledge can be used to capture the intended semantics in the *Māṇḍūkya Upaniṣad*.

Given a *jīva* j, the set of states associated with it (i.e., $\mathbb{S}_j$) can be further divided into $\mathbb{S}_j^D$ and $\mathbb{S}_j^A$ to represent the states in the dreaming and the waking world respectively. A person starting to dream is captured by the state transition $s_1 \xrightarrow{dream} s_2$ where $s_1 \in \mathbb{S}_j^A$ and $s_2 \in \mathbb{S}_j^D$. As dreaming can occur only when the *jīva* has a body, the property 'has*Sthūla*' needs to hold in both state s_1 and s_2.

In a dream state, the entities dreamt by the *jīva* j belong solely to j's space. Thus whenever $\mathbb{K}(j, o)(s)$ holds where $s \in \mathbb{S}_j^D$, the object o has to belong to j's state space. It is possible that o may correspond to an object in some other's *jīva*'s state but it itself has belong to j's state space.

For example, one may dream about Kṛṣṇa teaching Arjuna but both Kṛṣṇa and Arjuna are in the dreamer's state and are not the "real" Kṛṣṇa and Arjuna. The Kṛṣṇa in the dream corresponds to the real Kṛṣṇa but is not the real Kṛṣṇa. So in state s where *jīva* j is dreaming, we have $\mathbb{K}(j, teach(k, a))$ where the predicate $teach(k, a)$ indicates Kṛṣṇa is teaching Arjuna. But k and a are not the same as Kṛṣṇa and Arjuna and neither are k and a some random entities. The Kṛṣṇa and Arjuna in the dream do have a link to the real Kṛṣṇa and Arjuna.

To capture the relation between the entities in the dream state and in the real world, we define a map that links Kṛṣṇa with k and Arjuna with a. Formally, this map contains elements of the form (a, d_a^j) where a is an entity and d_a^j is a's appearance in j's dream. So if, K and A represent the real Kṛṣṇa and Arjuna, we will have (K, d_K^j) and (A, d_A^j). That is, k, the real Kṛṣṇa is mapped to d_K^j the Kṛṣṇa in the dream and a, the real Arjuna, is mapped to d_A^j the Arjuna in the dream.

The linking of state transitions and knowledge-based predicates using the above example is

illustrated below. Consider the following sequence of state transitions for an individual *jīva* j.

$s_1 \xrightarrow{dream} s_2$ representing j starts dreaming,
$s_2 \xrightarrow{teachingInDream} s_3$ where j starts dreaming about Kṛṣṇa teaching Arjuna
$s_3 \xrightarrow{awake} s_4$ and j wakes up.

In state s_3 we can state that $\mathbb{K}(j, teach(k, a))$ holds while in s_4 $\mathbb{K}(j, teach(k, a))$ will not hold. However if the person knows that Kṛṣṇa taught the Gītā to Arjuna, the knowledge formula $\mathbb{K}(j, teach(K, A))$ will hold. in states s_1 and s_4.

Another simple example is when a person (say *jīva j*) dreams about achieving something. The person dreaming (j) and the person in the dream (d_j^j, i.e., j is dreaming about j) are clearly not the same but are related. Thus j will be mapped to d_j^j.

This concludes the description of the various examples. In the next section, we will put some of these ideas together to construct a big-picture system view.

4. System View

Thus far we have looked at individual concepts that are used to explain the different metaphysical concepts in Vedānta. We now present a high-level system view without all the internal details of the individual systems. Figure 3 has a simple depiction of how the universe arises from *Brahman,* and an abstract semantics for *saṃsāra*, *pralaya* and *mokṣa*. The system has a potential unending cycle because after *pralaya* there is a re-creation of the universe. Figure 3 does not indicate how many times the cycle of *saṃsāra* is taken. The exact number of iterations would depend on the specific values of *karma* and the actions that update it. Thus the *karma* shown in the diagram is not a specific value. It represents the presence of *karma* for all concrete states associated with the *jīva*. So, this general description needs be instantiated for each particular situation, to explain how an individual's life unfolds.

The transition labelled *mokṣa* leading to *Brahman* is technically valid only in *advaita*. Also, it does not capture the behaviour of a *jīvan mukta*. Similarly, the label of *māyā* on the transition from *Brahman* to the universe is also specific to *advaita*. A dashed arrow is show from *Brahman* to the universe to illustrate that the state "*Brahman*" does not change. So the transition does not represent an evolution of *Brahman*'s state. Formally, this can also be represented by asserting that *Brahman* exists at each and every state.

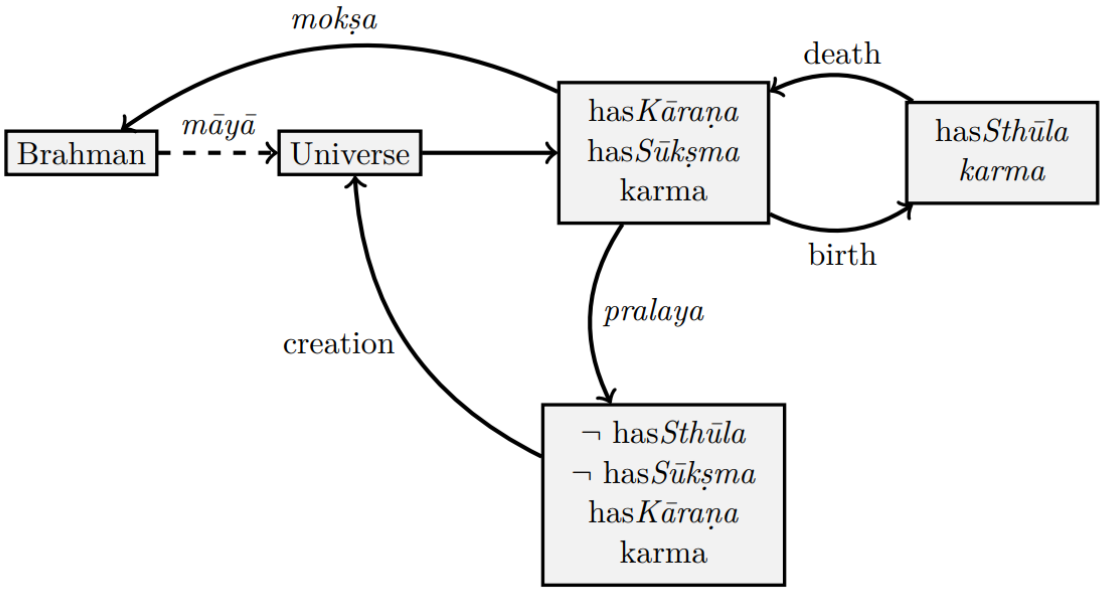

Figure 3: System Behaviour in *Advaita*

The transition system in Figure 3 can be interpreted as giving a precise semantics for the *mahāvākya*s [15] that appear in the different *Upaniṣad*s. For example, the statement "*tat tvam asi*" (or Thou are That) can be stated as follows. For each *jīva j* that corresponds to "Thou" in any state *s*, we can always find a path from *s* that leads to *Brahman*. So in our formalism we do not equate the true self of any *jīva* with *Brahman*. It is about the possible evolution of behaviour that can eventually reach *Brahman*. Thus the semantics of the *mahāvākya* in our system is that all *jīva*s can reach *Brahman*.

The statement "*prajñānam Brahmā*" requires a more careful analysis. The statement is not about any *jīva*. Hence it is not directly related to the transition system. As the statement is about knowledge, the semantic characterisation of *Brahman* is the relation $\mathbb{K}$. This relation $\mathbb{K}$ can be associated with *Brahman* because *Brahman* cannot be known in the conventional way but "knows" everything. All other states will have some item that is not known. That is, for every state $s \neq B$, there exists a formula $\mathbb{K}(i, o)$ such that $\mathbb{K}(i, o)$ does not hold at s. That is, entity i does not know object o in state s. We axiomatically equate *Brahman* B with $\mathbb{K}$. We do not wish to state that B knows *everything* as that could lead to logical contradictions.

The *dvaita* view is captured in Figure 4. Firstly, *mokṣa* is reaching *Brahman*'s or *Viṣṇu*'s abode (*Vaikuṇṭha*) and not merging with *Brahman*. The relationship between *Viṣṇu* and *Vaikuṇṭha* is captured by the transition labelled "lives" to indicate *Viṣṇu* lives in *Vaikuṇṭha*. In *dvaita*, grace of God is important. Hence apart from *karma*, we include a new predicate called 'getsGrace'. *Dvaita* does not believe in *māyā* but has a notion of *līlā* (God's non-selfish play). Therefore, the transition (again shown as a dashed line to show that *Brahman* does not change) from *Brahman* or *Viṣṇu* is now labelled *līlā*. Here again, each *jīva* has a sequence of transitions that lead to *Vaikuṇṭha*.

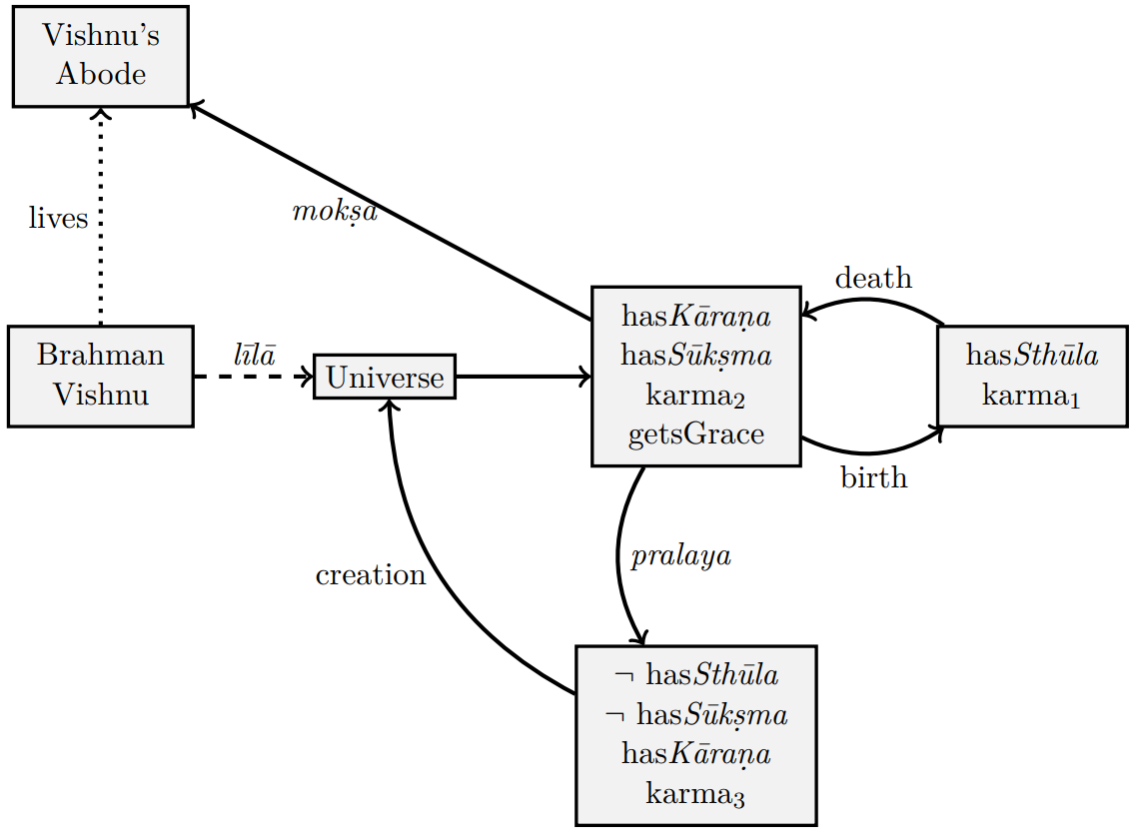

Figure 4: System Behaviour in *Dvaita*

Apart from identifying the difference in the definition of *mokṣa*, the formalism identifies what is common to *advaita* and *dvaita*. For instance, concepts such as *sthūla śarīra* and *pralaya* are not affected by the different interpretations of *mokṣa*.

5. Conclusion

In summary, this article has illustrated how many aspects of Hindu philosophy, viz., Vedānta can be captured in a mathematical framework. The key contributions are

- A set of states ($\mathbb{S}$) that can be partitioned for each *jīva* and within the states for the *jīva* it can be divided into dreaming and waking states. A map that can captures the correspondence between entities and their occurrences in people's dreams.
- A set of properties ($\mathbb{P}$) that can be used to describe the properties that hold at each state. This can be used to encode a variety of concepts including the *śarīra*s and *karma*. The different *śarīra*s are represented by simple predicates while *karma* is represented either as a simple pair of integers or as a pair of pair of integer values to capture *sañcita karma* and *prārabdha karma*. The notion of knowledge ($\mathbb{K}$) can also be associated with states to indicate what is known in each state.
- A set of transitions ($\rightarrow$) between states to capture behaviour. Transitions combined with properties such as 'godAllows(α)' for specific actions, enables the description of the role of God in free will for the *jīva*s. The transition system can also be used to define a *jīvan mukta* and when *videha mukti* can

occur.

- A set of interactions ($\mathbb{I}$) which are nothing but sets of transitions to capture joint behaviour. This is necessary as the *jīva*s interact with each other in this world. It is also useful to define the influence of others on a particular *jīva* and how that *jīva* reacts to this external influence. This set of possible interactions is used to identify a calm mind.
- Various examples (e.g., free will, aspects from the *Yogasūtra*s, *mokṣa*) to illustrate the usefulness of the formalism.

The formalism presented here can be used to describe concepts such as consciousness [11]. The formalism can also be extended to cover probabilistic behaviours and notions of information to accommodate other descriptions of consciousness [30]. Potential future work is to develop a deep semantics for specific concepts in Vedānta.

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